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Discussion of Errors in Gain Measurements of Standard Electromagnetic Horns

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ABSTRACT

In setting up a calibration service for measuring the gain of standard electromagnetic horns, one needs a reference horn in which one has developed a high degree of confidence. Although it is possible to calculate the gain of horns of certain design, confidence can be increased by carefully measuring the gain. This note examines a method for measuring the gain of two identical horns, listing the assumptions made in making such a measurement. The theory of 2-port waveguide junctions is applied to the analysis of the measurement technique. The method is shown to be essentially an attenuation measurement which has additional sources of error. Although these errors are not analyzed and evaluated in this note, the problem is perhaps more clearly stated than it was previously. The mismatch error in comparing two horns as receiving antennas is analyzed. Data is given on the aperture efficiency of standard horns which indicates that improvements in the design of such horns are feasible. It is concluded that, at present, an uncertainty limit of the order of tenths of decibels seems realistic, but hundredths of decibels seems unattainable until further refinements are made both in the standard horns themselves and in the measurement techniques.

Key Words: antennas, calibration, comparison, effective aperture, electromagnetic horns, gain, measurement, microwave, mismatch errors, Rayleigh distance, standard gain horns.

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OF STANDARD ELECTROMAGNETIC HORNS
R. W. Beatty

1. Introduction

In measuring the gain of an antenna such as an electromagnetic horn, comparison methods are widely used in which the gain relative to some accurately known standard antenna is determined. A standard horn may be constructed, following a certain design, and the gain may be calculated. Based upon comparisons between calculated and experimental values, the maximum uncertainty in this calculated gain has been estimated to be within ± 0.3 decibel [Slayton, 1954].

In order to increase one's confidence in the gain of the standard antenna, it is usually measured by a direct method. One well-known method [IEEE Standard, no. 149, 1965] employs two identical antennas, one used to transmit and the other used to receive. The gain is determined from the ratio of transmitted to received power, the aperture separation, and the wavelength.

If such a gain measurement is to be useful in increasing confidence in the calculated gain, the uncertainty in the measurement must lie within the limits ± 0.3 decibel. Although it is difficult to analyze the errors and be certain of the limits of uncertainty, this is necessary if accurate results are required.

In recent measurements in Canada [Jull and Deloli, 1964], an estimated probable error of ± 0.02 decibel was quoted for measurements at several frequencies in each of the bands 2.8 - 3.5 GHz, 6.0 - 7.0 GHz, and 12.4 - 14.5 GHz (Gc/s or 10^9 cycles per second). At the Bell Telephone Laboratories in the U.S.A. it was estimated

[Chu and Semplak, 1965] that errors from all sources caused a total uncertainty well within ± 0.1 decibel at a frequency of 4.08 GHz.

At the National Bureau of Standards, a calibration service for measuring gains of standard electromagnetic horns has not yet been established. However, some work has been done on the investigation of different techniques and measuring arrangements, and on the analysis of errors. The setting of final uncertainty limits must await the actual evaluation of limits of all of the various error components, and this has not been completed.

In the following, a version of the identical antenna method is briefly described and the assumptions made in a measurement are listed and discussed. The theory of 2-port waveguide junctions is applied[1] to analyzing the errors in the method caused by reflection (mismatch errors). The analysis is extended to obtain an expression for mismatch error in the comparison of two horns as receiving antennas.

The power gain g, (see G_1 in the list of symbols) is the directional gain or directivity d, multiplied by the efficiency η_T when transmitting. Thus the power gain includes the effect of dissipative losses in the antenna and requires that one define the physical boundaries of the antenna. One boundary is the aperture plane and the other is a terminal surface in the waveguide feed. This terminal surface may be arbitrarily chosen, but there is then a different power gain corresponding to each choice. In contrast,

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[1] The theory of 2-port waveguide junctions was also applied in the determination of antenna gain from scattering cross-section measurements by Garbacz [1964]. His technique is quite interesting but at present does not seem to promise high accuracy in gain measurement.

the directivity does not depend upon the location of this reference plane.

The directivity is sometimes defined as a function which depends upon the direction. In this note, d refers to the directivity in the axial direction in which the directivity function for standard horns is maximum.

The directivity as defined in the list of symbols is the directivity when transmitting. The directivity when receiving is defined in a similar way with reference to the receiving pattern. If nonreciprocal elements are excluded, which is usually true for standard horns, the directivity is the same whether receiving or transmitting.

2. Brief Description of Method

The method as described in the [1965] IEEE Standard is modified slightly in order to spell out precisely what is measured and to facilitate a more rigorous analysis. Instead of the ratio of P_r to P_t (transmitting antenna with power P_t and the other receiving power of amount P_r), a ratio of initial to final load powers is used to determine the gain. In the initial condition, the load is connected directly to the signal source (alternately, connection may be made by means of a long section of waveguide to avoid the need for moving either signal source or load during a measurement), and in the final condition the signal source feeds the transmitting antenna and the load terminates the receiving antenna. The antennas are linearly polarized in the same direction and their main beams are colinear so that maximum power is received. The above conditions are indicated in figure 1. Standard electromagnetic horns are usually linearly polarized and once calibrated can be used in a comparison

method [Clayton and Hollis, 1966] to determine polarization char-
acteristics of other more complicated antennas.

The measurement of antenna gain is thus essentially a measure-
ment of insertion loss or more precisely a measurement of sub-
stitution loss [Beatty, 1964]. (The final 2-port is substituted in
place of the initial 2-port.) Note that the initial 2-port can represent
either a long section of waveguide and the two waveguide joints by
which it is connected to signal source and load, or it can represent
a single waveguide joint if the load is initially connected directly
to the signal source.

3. Assumptions Made

It is important to take a careful look at the assumptions made in
the measurement of antenna gain by this method, because violation of
any one of the assumptions may cause some uncertainty in the result.
The following list may not be complete, but is intended to include
all of the basic assumptions.   Note that it would be very difficult to
satisfy conditions 4 and 5 simultaneously and hence, compromises
are usually made.

1. Sinusoidal time variation of the electromagnetic field
quantities E & H. (Modulation may exist, but is an
undesired complication.) Strictly, this implies a single
frequency. (The spectral width of the signal source
should be quite narrow.)

2. Single mode propagated in waveguides at terminal surfaces
1 and 2 (see figure 1).

3. Medium is lossless, linear, reciprocal, and isotropic.
(Normally, these conditions are closely approximated by
air.)

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4. The antennas are in free space (there are no surrounding objects). It usually requires considerable effort and expense to closely approximate this condition.

5. The antennas are sufficiently separated so that:
   a) the receiving antenna is in the far field (static field and induction field are negligible),
   b) the unperturbed field at the receiving location is essentially uniform in amplitude and phase over the aperture (in addition to sufficient separation, the free space condition is helpful), and
   c) the energy scattered by the antennas produces negligible effects.

6. The antenna separation r is clearly defined. (This is facilitated if the apertures are planar and perpendicular to the antenna axes.)

7. The antennas are linearly polarized and are oriented so that their axes are colinear; their directions of polarization are the same; and their apertures face one another and would coincide if brought together without rotation about the axis.

8. The signal source and receiver have the same characteristics for the initial condition as for the final condition of the measurement. (This requirement is satisfied if they are stable during the measurement.)

9. The signal source and load are non-reflecting at their terminal surfaces 1 and 2 as in figure 1 $\left(\Gamma_G = \Gamma_L = 0 \right).$

10. The antennas are linear, reciprocal, and (especially with regard to dimensions) stable.

11. The antennas are identical. (This is not required if a third antenna is employed.)

4. Minimum Antenna Separation

It is probably best to determine the effects of insufficient antenna separation by making a series of gain measurements with the separation gradually varied in steps [Jakes, 1951]. However, one can calculate a "minimum separation" based upon variation of phase of the plane wave front over the aperture of the receiving antenna. This is useful in designing a test arrangement.

The phase of the wave reaching the edge of the aperture of the receiving antenna is delayed from that reaching the center because of the extra length δ, which the wave must travel. This is shown in figure 2. Depending upon what phase variation is considered permissible, one may derive a relationship for the "minimum separation" r_m. If $\delta = \dfrac{\lambda}{4}$,

$r_m = \dfrac{a^2}{2\lambda}$, and is called the "Rayleigh Distance" [Hansen, 1964]. It tends to separate the Fresnel and Fraunhofer diffraction regions (in optics). A minimum separation of $\dfrac{2a^2}{\lambda}$ has been used, but it was recognized that this did not put the receiving antenna far enough into the Fraunhofer region. Braun [1953] found that for a separation of $4\dfrac{a^2}{\lambda}$, the measured gain was 0.2 decibel below the correct value (16 decibels) and for $8\dfrac{a^2}{\lambda}$, 0.1 decibel below. The amount depended upon the

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gain measured. He published calculated curves for correcting the measured gains of pyramidal horns. Thus a convenient separation may be chosen and a correction applied to the measured result. [2]

A graph is shown in figure 3 which gives the minimum separation $r_m = 10 \dfrac{a^2}{\lambda}$ in feet (20 times the Rayleigh distance), vs. the operating frequency in GHz for antennas of different gains. The gains (G_0) are based upon a circular aperture with uniform field distribution. In practice, an antenna with the same gain will have a larger aperture, so that the minimum distances obtained from the graph should be multiplied by the ratio of the apertures, which in practice will give factors from 2-5.

Instead of the minimum separation as determined from figure 3, a separation giving a convenient insertion loss may be desired. Losses of approximately 10-30 decibels are convenient to measure and correspond (at 10 GHz) approximately to the separations shown in figure 4.

It can be seen from figure 3 that at 10 GHz, a separation of 12 feet is 20 times the Rayleigh distance if G_0 is 21 decibels. An actual antenna with the same aperture size might have a gain of 18 decibels. (See Appendix C for a discussion of effective aperture areas of actual antennas.) It can be seen from figure 4 that the measured

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[2] According to Chu and Semplak, Braun's assumptions about the received power are questionable, since the power in the transmitted wave was averaged over the receiving aperture. Using Hu's transmission formula, [Hu, 1958], they have employed a digital computer to calculate correction ratios for the far-zone gain of pyramidal horns measured at relatively short distances. Their measured gain of 20.11 decibels at 4.08 GHz agrees closely with the calculated gain of 20.15 decibels.

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insertion loss at 10 GHz will be approximately 27 decibels.

The curves of figures 3 and 4 are useful in making rough estimates but are not intended for use in accurate gain measurements.

5. Analysis of Gain Measurement

Consider the measurement arrangement of figure 1, and assume that the initial 2-port is such that $^i a_1 = {}^i b_2$ and $^i b_1 = {}^i a_2$. In the ideal case in which the generator and load are non-reflecting the measured substitution loss L_S is the attenuation of the final 2-port:

$$L_S = 10 \log_{10} \left(\frac{^i P_L}{^f P_L} \right) , \tag{1}$$

where superscripts i and f refer to initial and to final conditions, respectively.

Note that

$$\frac{^i P_L}{^f P_L} = {}^i \eta_1 \cdot \frac{^i P_1}{^f P_1} \cdot \frac{^f P_1}{^f P_L} = {}^i \eta_1 \; {}^f \eta_1 \frac{^i P_1}{^f P_1} . \tag{2}$$

The first factor on the right in (2) is the efficiency of the initial 2-port. It is very nearly unity if the load is initially connected directly to the signal source. However if a long section of waveguide is used to initially connect the source and load, then $- 10 \log_{10} \left({}^i \eta_1 \right) = \alpha \ell$, where $\alpha \ell$ is the attenuation in decibels of the section of waveguide having a length ℓ.

The second factor on the right in (2) is the efficiency of the final 2-port. It can be expressed in terms of the antenna parameters[3]

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[3] See Appendices A and B for discussion of effective aperture relationships.

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(gain and reflection coefficients), the antenna separation r, and the wavelength λ as follows, making reference to figure 1 and to the list of symbols given later, and assuming that $\Gamma_G = \Gamma_L = 0$. It is also assumed that the antennas are separated far enough so that interaction and diffraction effects can be safely neglected:

$$
{}^{f}P_L = S_2 A_{2M} \left(1 - |S_{22}|^2 \right) = \frac{\Phi_1 A_{2M}}{r^2} \left(1 - |S_{22}|^2 \right)
$$

$$
= \frac{d_1 P_{T1} A_{2M}}{4\pi r^2} \left(1 - |S_{22}|^2 \right) = \frac{d_1 \eta_{T1} {}^{f}P_1 A_{2M}}{4\pi r^2} = \left(1 - |S_{22}|^2 \right)
$$

$$
= \frac{g_1 A_{2M} {}^{f}P_1}{4\pi r^2} \left(1 - |S_{22}|^2 \right). \tag{3}
$$

But $\qquad A_{2M} = \dfrac{\lambda^2}{4\pi} g_2 ,$ $\qquad\qquad\qquad\qquad\qquad\qquad$ (4)

thus $\qquad \dfrac{{}^{f}P_L}{{}^{f}P_1} = g_1 g_2 \left(\dfrac{\lambda}{4\pi r} \right)^2 \left(1 - |S_{22}|^2 \right).$ $\qquad\qquad$ (5)

This is one form of the free-space transmission formula [Schelkunoff and Friis, 1952].

The third factor on the right in (2) is the (conjugate) mismatch loss. Assuming that for the initial 2-port ${}^{i}\Gamma_1 = \Gamma_L = 0$, the mismatch loss can be calculated from [Beatty, June, 1964]

$$
\frac{{}^{f}P_1}{{}^{i}P_1} = 1 - |S_{11}|^2 , \tag{6}
$$

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where $S_{11} = {}^f\Gamma_1$ under the condition $\Gamma_L = 0$. For antenna No. 1 radiating into free space, $|S_{11}|$ is related to its VSWR σ_1 by

$$|S_{11}| = \frac{\sigma_1 - 1}{\sigma_1 + 1}. \tag{7}$$

The reciprocal of (2) is then written as follows:

$$\frac{{}^fP_L}{{}^iP_L} = \frac{g_1 g_2}{\eta_1} \left(\frac{\lambda}{4\pi r}\right)^2 \left(1 - |S_{11}|^2\right)\left(1 - |S_{22}|^2\right). \tag{8}$$

The measured substitution loss is then given by substituting (8) in (1). If the antennas are identical, $g_1 = g_2 = g$, and $S_{11} = S_{22} = \Gamma_A$. The measured substitution loss is then

$$L_S = -20 \log_{10} g + 20 \log_{10}\left(\frac{4\pi r}{\lambda}\right) + 20 \log_{10}\left(\frac{1}{1 - |\Gamma_A|^2}\right) - \alpha\ell. \tag{9}$$

The gain is written

$$G = 10 \log_{10}\left(\frac{4\pi r}{\lambda}\right) + 10 \log_{10}\left(\frac{1}{1 - |\Gamma_A|^2}\right) - \frac{\alpha\ell}{2} - \frac{L_S}{2}. \tag{10}$$

In order to obtain the expression given in the [1965] IEEE Standard, one must assume that the antennas are non-reflecting ($\Gamma_A = 0$), and that the load is initially connected directly to the generator ($\alpha\ell = 0$).

It is certainly possible in principle to incorporate a tuner in the antenna and adjust it at the operating frequency to achieve the condition $\Gamma_A = 0$, but this may not always be desirable for one reason or another. Thus (10) should be used to calculate the gain from the measured substitution loss. If the antenna reflections are appreciable but are neglected in the calculation of gain, the result will be too low by the amount

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$$C_\sigma = 10 \log_{10} \left(\frac{1}{1 - |\Gamma_A|^2} \right) = 10 \log_{10} \frac{(\sigma + 1)^2}{4\sigma}. \qquad (11)$$

A graph of the mismatch correction according to (11) is given in figure 5.

If the signal source and load do not satisfy the conditions $\Gamma_G = \Gamma_L = 0$, the mismatch error can be evaluated by the same methods used in attenuation and insertion loss measurements. At the present time, errors from connectors or waveguide joints at the insertion point are probably small compared with other components of error. The analysis of their effect on this type of measurement has been published [Beatty, December 1964] and is available if needed.

If the two antennas are not identical, the gains g_1 and g_2 may still be determined by introducing a third antenna having a gain g_3.

Note that the product $g_1 g_2$ can be determined from (8). In a similar way, the products $g_1 g_3$ and $g_2 g_3$ can be determined from measurements like the one described, using equations similar to (8). We can then calculate the ratio of g_1 to g_2 from the measured products $g_1 g_3$ and $g_2 g_3$. Thus

$$g_1 = \sqrt{\frac{g_1 g_3}{g_2 g_3} \cdot g_1 g_2}, \quad \text{and} \qquad (12)$$

$$g_2 = \sqrt{\frac{g_1 g_2}{g_1 g_3} \cdot g_2 g_3}. \qquad (13)$$

It is desirable to closely approach the condition $g_1 = g_2 = g_3$ so that additional errors introduced by the above procedure can be kept small.

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Depending upon the separation used and the accuracy required, it may be necessary to make a number of gain measurements with different spacings so that one can determine the interaction effect and average it out. In principle, one can incorporate a tuner into the antenna and adjust it so as to eliminate or at least minimize scattered radiation from the receiving antenna in the direction along the axis. If this were done, the interaction effect could be practically eliminated. This technique would be expected to be less effective for shorter separations because one cannot eliminate scattering in all directions by this technique, [Harrington, 1964] and the transmitting antenna would subtend a larger angle if the separation were decreased.

One can choose a separation greater than the Rayleigh distance in making gain measurements and this places the receiving horn in the Fraunhofer region. However this does not free one completely of Fresnel region effects. For example the variation of phase across the aperture of the receiving antenna due to path length differences does not cease as one goes into the Fraunhofer region, but only diminishes. Various authors [for example, Soejima, 1963] have dealt with methods of correcting the near field gain measurements to obtain the far field gain. In most cases they assume a certain aperture illumination or type of horn design.

6. Analysis of the Comparison Technique

Once we have determined the gain of a standard antenna and have developed maximum confidence in the result, it may be used in a comparison technique to calibrate other standard antennas. In one version of this technique, an antenna system is used such as in figure 1,

and the antenna to be calibrated is simply substituted in place of the
reference antenna which is in the receiving position, keeping every-
thing else unchanged.

To a good approximation, the ratio of the powers absorbed by the
load under these two conditions equals the ratio of the antenna gains.
If the power ratio is determined, and the gain of the reference antenna
is known, we can calculate the gain of the antenna to be calibrated.

If we use this approximate relationship to determine the gain,
the result will be in error because of the interaction of load and
antenna reflections. The amount of this mismatch error may be
determined as follows.

The power absorbed by a load connected to the receiving antenna
is $P_L = S A$, and (see Appendix B)

$$A = A_M \frac{\left(1 - |\Gamma_A|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - \Gamma_A \Gamma_L|^2} . \qquad (14)$$

But $\qquad A_M = \frac{\lambda^2}{4\pi} g$,

thus $\qquad P_L = g S \frac{\lambda^2}{4\pi} \cdot \frac{\left(1 - |\Gamma_A|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - \Gamma_A \Gamma_L|^2} . \qquad (15)$

If we substitute the antenna to be calibrated (antenna C) in place
of the reference antenna (antenna R), and do not change the load, the
ratio of powers P_{LC} to P_{LR} absorbed by the load is

$$\frac{P_{LC}}{P_{LR}} = \frac{g_C}{g_R} \frac{1 - |\Gamma_C|^2}{1 - |\Gamma_R|^2} \cdot \left|\frac{1 - \Gamma_R \Gamma_L}{1 - \Gamma_C \Gamma_L}\right|^2 , \qquad (16)$$

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where the subscripts R and C refer to antennas R and C, respectively.

The gain G_C of the antenna being calibrated is

$$G_C = G_R + 10 \log_{10}\left(\frac{P_{LC}}{P_{LR}}\right) + 10 \log_{10}\left(\frac{1 - |\Gamma_R|^2}{1 - |\Gamma_C|^2} \cdot \left|\frac{1 - \Gamma_C \Gamma_L}{1 - \Gamma_R \Gamma_L}\right|^2\right). \quad (17)$$

The last term on the right is the mismatch error. In order to evaluate it completely, one would need to know both phases and magnitudes of Γ_R, Γ_C, and Γ_L. However, if only the magnitudes were determined by VSWR measurements, one could calculate bounds between which this error must lie. A similar approach has been used for mismatch errors in power measurements [Beatty and MacPherson, 1953] and the same relationships apply.

Usually the load reflection coefficient Γ_L may be made small by the adjustment of appropriate tuners. If the load is non-reflecting $\left(\Gamma_L = 0\right)$, (17) becomes

$$\left(G_C\right)_{\Gamma_L = 0} = G_R + 10 \log_{10}\left(\frac{P_{LC}}{P_{LR}}\right) + 10 \log_{10} \frac{1 - |\Gamma_R|^2}{1 - |\Gamma_C|^2} . \quad (18)$$

The mismatch error term is now easier to evaluate, requiring only the VSWR's of the two antennas.

If it is permitted to tune antennas R and C for the condition $\Gamma_R = \Gamma_C = 0$, the mismatch term will ideally vanish, and we will have the simple relationship

$$\left(G_C\right)_{\Gamma_L = \Gamma_R = \Gamma_C = 0} = G_R + 10 \log_{10}\left(\frac{P_{LC}}{P_{LR}}\right) . \quad (19)$$

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Such a tuning would be frequency sensitive and would require readjustment each time a new frequency was considered. Thus it might not be used in some cases.

The ratio of P_{LC} to P_{LR} in decibels may be determined by connecting a calibrated variable attenuator between the signal source and the transmitting antenna and adjusting it to return the load power to the same value after substituting antenna C for antenna R as described. The change in attenuation equals the desired power ratio in decibels.

It is apparent that this comparison technique is subject to fewer sources of error than the identical antenna method, but many of the same assumptions are made, particularly with regard to antenna separation and free-space conditions.

7. Conclusions

The well-known method discussed in this note is recommended for use in measuring the gain of a standard electromagnetic horn which would then be used as a reference in calibrating other horns in a comparison process. It is anticipated that the comparison technique would be simpler and subject to less error than the method discussed here.

One can justify greater effort in measuring the reference horn than in a method to be used for routine calibrations of many horns. Thus it is worthwhile to take a close look at the sources of error which are present in this method.

Since the measurement of gain by this technique is essentially an attenuation measurement, it is subject to the usual errors one would encounter in measuring the attenuation of a fixed pad plus additional errors which arise due to violation of the assumptions

made. In particular, errors due to violation of the free-space condition and the uniform plane wave at the receiving antenna aperture are difficult to evaluate and should receive greater attention. At present, one has difficulty in measuring the attenuation of a fixed pad to an uncertainty less than 0.5 percent of the attenuation value, or 0.05 decibel, whichever is greater. It would therefore seem difficult to do any better with measurements of antenna gain. It appears that the design of horns intended for use as standard horns should be improved, to reduce their scattering and their VSWR.

Reduction of scattering would reduce errors due to multiple reflections between antennas. (Interaction effects.) Reduction of the VSWR would not only reduce the error or correction mentioned in this note, but would reduce the mismatch error in the "attenuation" measurement. VSWR's of commercially available standard gain horns range from 1.15 to 1.25. A goal of 1.02 or better would be worthwhile for standard horns. At present, pyramidal horns, for which the gain can be calculated, and for which corrections are available for near field measurements of the far field gain, do not seem to have the best VSWR and scattering characteristics. (See Appendix C.) In the comparison technique, mismatch errors should be considered. They can be eliminated by making the load and the antennas non-reflecting.

8. Acknowledgement

9. List of Symbols

$^i a_1$ -   Amplitude of voltage wave at port 1 incident upon initial 2-port waveguide junction

$^f a_1$ -   Amplitude of voltage wave at port 1 incident upon final 2-port waveguide junction

$^i b_1, {}^f b_1$ -   Amplitudes of emergent voltage waves at port 1 corresponding to $^i a_1$ and $^f a_1$ above

$^i a_2$ -   Amplitude of voltage wave at port 2 incident upon initial 2-port waveguide junction

$^f a_2$ -   Amplitude of voltage wave at port 2 incident upon final 2-port waveguide junction

$^i b_2, {}^f b_2$ -   Amplitudes of emergent voltage waves at port 2 corresponding to $^i a_2$ and $^f a_2$ above .

Γ_G -   Voltage reflection coefficient of equivalent source at port 1

Γ_L -   Voltage reflection coefficient of equivalent load at port 2

$^i P_1, {}^f P_1$ -   Net power input at port 1 to initial and final waveguide junctions, respectively

$^i P_L, {}^f P_L$ -   Power absorbed by load under initial and final conditions, respectively

P_{T1} -   Total power radiated by antenna no. 1

Φ_1 -   Power radiated per unit solid angle in axial direction

S_2 -   Unperturbed radiated power flux per unit area at aperture of antenna no. 2 $\left(S_2 = \dfrac{\Phi_1}{r^2} \right)$

r   -   Distance between apertures of transmitting and receiving antennas

η_{T1} -   Efficiency (ratio of P_{T1} to $^f P_1$) of antenna no. 1 when transmitting

D_1 -   Directional gain (expressed in decibels) in axial direction for antenna no. 1. $D_1 = 10 \log_{10} d_1$, where d_1 is the ratio of the radiation intensity Φ_1 to the average radiation intensity $\left(P_{T1} \div 4\pi \right)$. It can be determined by integration of the radiation pattern.

Γ_{2i} -   Voltage reflection coefficient of equivalent source at port 2

G_1 -   Power gain (expressed in decibels) in axial direction for antenna no. 1. $G_1 = 10 \log_{10} g_1$, $g_1 = \eta_{T1} d_1$. (Referred to an isotropic radiator) η_{T1} is the efficiency (ratio of total radiated power to the net power input) of antenna no. 1 when transmitting.

A   -   Effective aperture area of an antenna. It is the power absorbed by a load connected to that antenna divided by the unperturbed power density S at the antenna aperture plane.

A_M -   Maximum aperture area of antenna no. 2. It is the power delivered to a conjugately matched load connected at terminal surface no. 2 divided by the unperturbed power flux per unit area S_2. (Incident plane wave linearly polarized and antenna oriented for maximum received signal)

σ — Voltage standing-wave ratio

S_{11} — Reflection coefficient of antenna no. 1 when radiating into free space

λ — Wavelength of unperturbed plane wave

α — Attenuation constant of waveguide expressed in decibels per unit length

L_S — Substitution loss, the ratio of powers $^{i}P_L$ to $^{f}P_L$ absorbed by the load when the final 2-port is substituted for the initial 2-port between the signal source and the load.

The above list of symbols and definitions is in harmony with the IEEE Standard No. 149 [January 1965] "Test Procedures for Antennas."

10. Appendix A

10.1 Relationship of Effective Aperture Area to Gain

The effective aperture area A is a convenient quantity which gives the power absorbed by a load connected to the receiving antenna if one knows the unperturbed power density S at the aperture plane. It follows that A depends upon where one chooses the terminal surface to separate the receiving antenna and its load. It is apparent that A also depends upon the angle between the polarizations of the incident wave and of the receiving antenna, and upon the load and antenna impedances. A complete definition has been given by Tai [1961]. However in this note, it is assumed that the polarizations are aligned for maximum received power, and that the load impedance is the conjugate of the antenna impedance. Under these conditions, the effective aperture area is maximum A_M, as noted in Appendix B.

The relationship $A_M = \dfrac{\lambda^2}{4\pi} g$ can be derived in two steps:

(1) The relationship $\dfrac{A_1}{A_2} = \dfrac{g_1}{g_2}$ is derived for any two (reciprocal) antennas.

(2) The relationship $A_M = \dfrac{\lambda^2}{4\pi} g$ is derived for a particular kind of antenna - a thin half-wave dipole.

This derivation has value here because the assumptions and necessary conditions are brought out.

In the first derivation, consider the 2-port representation shown in figure 1. The efficiency ${}^f\eta_1$ of the final 2-port is

$$ {}^f\eta_1 = \frac{g_1 A_2}{4\pi r^2}\left(1 - |S_{22}|^2\right)\ . \tag{20} $$

If we interchange the generator and load, the efficiency ${}^f\eta_2$ of the final 2-port is

$$^{f}\eta_2 = \frac{g_2 A_1}{4\pi r^2} \left(1 - |S_{11}|^2\right) . \tag{21}$$

It follows that

$$\frac{A_1}{A_2} = \frac{g_1}{g_2} \cdot \frac{\eta_2}{\eta_1} \cdot \frac{1 - |S_{22}|^2}{1 - |S_{11}|^2} , \tag{22}$$

where the superscript f has been dropped for simplicity. Note that the desired relationship between the effective aperture areas and the gains is obtained if $\eta_1\left(1- |S_{11}|^2\right) = \eta_2\left(1 - |S_{22}|^2\right)$. To investigate further, we write η_1 and η_2 in terms of the scattering coefficients of the 2-port as follows [Kerns and Beatty, 1967]:

$$\eta_1 = \frac{Z_{01}}{Z_{02}} \cdot \frac{|S_{21}|^2}{1-|S_{11}|^2} \tag{23}$$

$$\eta_2 = \frac{Z_{02}}{Z_{01}} \cdot \frac{|S_{12}|^2}{1-|S_{22}|^2} . \tag{24}$$

The ratio is

$$\frac{\eta_1}{\eta_2} = \left(\frac{Z_{01}}{Z_{02}}\right)^2 \cdot \left|\frac{S_{21}}{S_{12}}\right|^2 \cdot \frac{1-|S_{22}|^2}{1-|S_{11}|^2} . \tag{25}$$

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If nonreciprocal elements are excluded from the 2-port, then the condition $S_{12}Z_{02} = S_{21}Z_{01}$ holds [Kerns and Beatty, 1967]. It follows that $\eta_1 \left(1 - |S_{11}|^2\right) = \eta_2 \left(1 - |S_{22}|^2\right)$, and

$$\frac{A_1}{A_2} = \frac{g_1}{g_2} . \tag{26}$$

In the second derivation consider a thin half-wave dipole for which the impedance Z at the center is [Kraus, 1950]

$$Z_A = 30 \left[S_1 (2\pi) + j \, Si \, (2\pi) \right] = R_A + j \, X_A$$

$$Z_A = 30 \left[\gamma + \ell n \, (2\pi) - Ci \, (2\pi) + j \, Si \, (2\pi) \right]$$

$$Z_A = 73.13 + j \, 42.5 \text{ ohms.} \tag{27}$$

The directive gain can be shown to be

$$d = \frac{120}{R_A} = 1.641. \tag{28}$$

The maximum effective aperture area can be shown to be

$$A_M = \frac{30 \, \lambda^2}{\pi \, R_A} = \frac{\lambda^2}{4\pi} d = \frac{\lambda^2}{4\pi} g . \tag{29}$$

This holds for the half-wave dipole, and because of (26), it holds for any (reciprocal) antenna.

In addition to accounting for the power absorbed by the load, one can account for the scattered power SA_s and the power dissipated in the antenna SA_d by defining other "aperture areas" A_s and A_d [Kraus, 1950]. This is feasible in the absence of multiple reflections, but is of questionable value in a rigorous analysis since it might encourage unduly simple (and incorrect) interpretations of the actual situation. One also finds in the literature an effective aperture area defined as

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$\lambda^2 d/4\pi$, where d is the directivity. This is of questionable value because it gives the available power (power delivered to a conjugately matched load) from a lossless antenna having the same impedance and directivity as the actual antenna.

Kraus [1950] has defined a collecting aperture A_c and the physical aperture A_p, where $A_c = A_M + A_s + A_d$, and A_p is the physical area of the mouth of the horn. Presumably, all but A_p could be affected by the load connected to the antenna and it is understood that the same load, a conjugately matched load, applies to each.

It might be useful to define a scattering cross-sectional area A_{sc} corresponding to a non-reflecting load ($\Gamma_L = 0$) connected to the antenna. It would be characteristic of the antenna itself since none of the scattered power could be considered as re-radiated upon reflection from the load. This scattered power would have a different "radiation pattern" from that of the horn when transmitting while the power re-radiated upon reflection would have the same radiation pattern.

11. Appendix B

11.1 Effective Aperture Areas of Arbitrarily Terminated Antennas

It can be shown as follows that the effective aperture area A
of an antenna terminated in a load having a voltage reflection coefficient
Γ_L equals

$$A = A_M \ \frac{\left(1 - |\Gamma_A|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - \Gamma_A \Gamma_L|^2} \ , \tag{30}$$

where A_M is the maximum effective aperture area (obtained when the
load impedance provides a conjugate match to the antenna impedance).

Equivalent expressions have been derived by C. T. Tai [1961],
whose impedance mismatch factor g (not to be confused with power gain
g in this paper) is expressed in terms of impedances, and G. Borgiotti
[1964], whose formula for the equivalent area of an antenna is essentially
the same as (30) except that he uses the directive gain d instead of the
power gain g.

Consider the Thévenin equivalent circuit of a receiving antenna
as shown in figure 9. It can be shown [Kerns and Beatty, 1967] that the
power delivered to the load is

$$P_L = \frac{e^2}{4 Z_0} \cdot \frac{\left(1 - |\Gamma_A|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - \Gamma_A \Gamma_L|^2} \ , \tag{31}$$

where Z_0 is the real characteristic impedance of the waveguide by means
of which the load is connected to the antenna. When the load is conju-
gately matched to the antenna, $R_L = R_A$ and $X_L = -X_A$, or $\Gamma_L = \Gamma_A^*$.
Then

$$P_M = \frac{e^2}{4 Z_0} \ . \tag{32}$$

The effective aperture area is defined so that $P_L = SA$. It is apparent that the unperturbed power density S is not changed by changing Γ_L. Thus

$$\frac{P_{L1}}{P_{L2}} = \frac{A_1}{A_2} \text{, or } A_1 = \frac{P_{L1}}{P_{L2}} \cdot A_2 \quad . \tag{33}$$

If P_{L2} is the available power P_M and A_2 is the maximum effective area A_M, then

$$A = \frac{P_L}{P_M} A_M \text{,} \tag{34}$$

and (30) follows upon manipulation of (31), (32), and (33). Since P_L can never be greater than P_M, it follows that A cannot be greater than A_M.

12. Appendix C

12.1 Effective Aperture Areas of Electromagnetic Horns

It is interesting to compare the effective aperture areas of electromagnetic horns to their actual aperture areas. This was done for some commercially available standard gain horns and the results are shown in figures 6 and 7. The effective aperture areas were obtained by multiplying the power gain by $\lambda^2/4\pi$ in each case.

Notice in figure 6 that horn A has a higher aperture efficiency (effective aperture area divided by actual aperture area) than horn B. The designs of the horns are quite different as shown in the (to scale) sketches. Apparently, the smoothly tapered throat of horn A reduces scattering and improves the aperture efficiency. The reduction of scattering is desirable in a standard gain horn because this reduces interactions between antennas when they are calibrated in the manner discussed in this note.

Minimum scattering closely corresponds to maximum aperture efficiency when the dissipative losses are relatively small. This in turn corresponds to maximum gain for a given aperture area. The maximum gain is obtained when the aperture illumination is uniform. Hence, a design principle for reducing scattering is to try to produce constant-phase, uniform illumination of the aperture when transmitting. This will result in reduced scattering when the antenna is receiving.

The aperture efficiency characteristic of horn C shown in figure 7 is similar to that of horn B, which has a similar design.

The gain of horn B relative to G_0, the gain of a horn of the same actual aperture area but having uniform illumination, is shown in figure 8. It is seen that the gain of the actual horn is 3 to 5 decibels lower than the maximum gain possible for a horn having the same aperture area. It is suspected that the discrepancy is caused mostly by scattering (over half

the incident power is scattered) and perhpas less than 0.2 decibel is caused by dissipation. There is as yet no direct evidence to support this viewpoint, but it seems reasonable.

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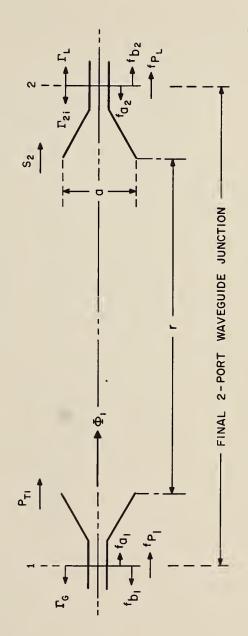
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15. Figure Captions

Fig. 1   Idealized model representing system of transmitting and
receiving antennas. Initially the source and receiver may
be connected together directly, or by a long section of wave-
guide, represented by the initial 2-port waveguide junction.
Finally, this may be replaced by the antenna system as
shown. The meanings of the symbols are given in a list in
the note.

Fig. 2   Diagram illustrating variation of phase of radiation over the
aperture of the receiving antenna. The minimum separation
r_m corresponds to the maximum permissible change δ in
path length.

Fig. 3   Minimum separation $r_m = 10 \dfrac{a^2}{\lambda}$ of antennas of gain G_0.
a = largest aperture dimension, and G_0 = gain of antenna
having uniform illumination over circular aperture of
diameter a.

Fig. 4   Calculated attenuation of send-receive system of identical
antennas versus separation of antennas in feet, for antennas
having gains from 15-30 decibels.

Fig. 5   Calculated antenna mismatch correction to gain in decibels
versus magnitude of antenna reflection coefficient.

Fig. 6   Aperture efficiencies versus frequency of two commercially
available standard gain horns at X-band frequencies
(8.2 - 12.4 GHz).

Fig. 7   Aperture efficiency versus frequency of a commercially
available, pyramidal design, standard gain horn at K-band
frequencies (26.5 - 40 GHz).

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FIG. I

FIG. 2

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$$\left(\frac{r_m}{\lambda}\right) = 10\left(\frac{a}{\lambda}\right)^2 \qquad G_0 = \left(\frac{\pi a}{\lambda}\right)^2$$

FREQUENCY - GHz

FIG. 3

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SUBSTITUTION LOSS – DECIBELS

$$L_s = 20 \log_{10} \left(\frac{4\pi r}{\lambda} \right) - 2 G$$

(ANTENNA GAIN)   G = 20 DECIBELS

15

25

30

r (SEPARATION BETWEEN ANTENNAS) – FEET

FIG. 4

SUBSTITUTION LOSS – DECIBELS

MAGNITUDE OF ANTENNA REFLECTION COEFFICIENT $|\Gamma_A|$

FIG. 5

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APERTURE EFFICIENCY

FREQUENCY – GHz

HORN "A"

HORN "B"

APERTURE EFFICIENCY

FIG. 6

APERTURE EFFICIENCY

FREQUENCY - GHz

FIG. 7

HORN "C"

FIG. 8

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FIG. 9

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